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Game Theory for Global Climate Policy

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Abstract

This paper solves the global cap-and-trade game exactly as the public-goods game is normally solved and finds a problematic outcome. Abatement of greenhouse gas emissions is a global public good, and supplying a public good is a game with strong incentives to free ride. Adding a cap-and-trade mechanism to such a game makes the outcome less cooperative. It reduces total global abatement and polarizes the commitment levels of large and small countries.

Cap and trade also increases polarization of commitment levels that reflect differences in national incomes. Countries with relatively high pre-cap abatement levels commit to abate still more when a cap-and-trade policy is added. Low-abatement countries choose targets below their pre-cap abatement levels. The poorest countries and those with the lowest emissions adopt caps above business-as-usual emissions. These results parallel the current polarization of developed and developing countries.

As an alternative to the cap-and-trade mechanism, a global-price-target mechanism is considered. In the stylized form analyzed here, countries vote for a global price target and the lowest vote wins. Posting bonds makes the outcome self enforcing, and all countries price carbon at the target level. In a simple public-goods model, all countries vote for the optimal price target. In a more complex world with rich and poor countries, cooperation requires the introduction of a Green-Fund incentive. Linking its level to the level of the adopted price target induces poor countries to vote for the same target as rich countries. The outcome is quite cooperative and nearly efficient.

Contents

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1 Introduction

This paper concerns global cap and trade but not national cap and trade. The two are often confused but are entirely different. The difference stems from that fact that there is no world government to impose a global cap the way national governments impose a cap on a national cap-and-trade market. *This paper does not criticize national cap-and-trade programs*.

Without any global climate policy it still makes sense for countries to abate greenhouse gas emissions. However, because they capture only a fraction of the benefit from such abatement, they will abate too little for the global public good. Large countries will abate more relative to their size because they capture a larger fraction of the benefits they create.

Of course, countries could abate more than their simple self interest dictates in hopes of leading others to follow their example. A large number of experiments indicates this is not a particularly effective strategy and that the outcome of public-goods games tends to become less cooperative as they are played repeatedly. The trend over time is toward behavior that is not too far from the Nash equilibrium of a one-shot public goods game.

For this reason, and to keep the analysis tractable, I will focus on these simple Nash equilibria. It should be noted that the Nash strategies of the games considered are dominant strategies, which means that they are not just best considering the Nash strategy of the other players, but are best no matter what the other players choose to do. Consequently, these Nash equilibria are particularly convincing as predictions of behavior.

The games considered are usefully viewed as consisting of two parts, a global economy and a climate policy. Several of each are considered and they are matched in various combinations. Two important games consist of a global economy in which countries differ only by size coupled with either no-policy or with a global cap-and-trade policy. This allows examination of the basic effect of introducing a cap-and-trade policy. The result is three theorems which show that cap-and-trade results in (1) a price that is N times too low, where N is the number countries, (2) a reduced level of total abatement, and (3) a polarization of commitment levels. Low-commitment countries adopt lower, possibly negative abatement targets, and high-commitment countries adopt targets above their abatement levels absent cap and trade.

The second half of the paper examines a pair of polices that have been design to favor cooperation—a global carbon price target and a Green-Fund incentive. In the same simple economy in which cap-and-trade results in a price that is N times too low, a price-target, even without a Green-Fund incentive, results in the unanimous selection of the optimal price. In the real world, however, countries also vary by income and emissions per capita, and not only by size. If low income countries tend to abate less, this causes problems for both cap-and-trade, which again polarizes the commitment levels, and for the selection of a price target. Since the selection process

[*] The author is director of the Global Energy Policy Center, and would like to thank Peter Cramton and Roger Sparks for helpful discussions of the games presented here.

gives priority to the country desiring the lowest target, the Green-Fund incentive is required to overcome the reluctance of poor countries.

A simple Green-Fund incentive is introduced and coupled with an economy that exhibits both the free-rider problem and, among low-emission countries, a distaste for abatement that increases as emissions per capita decline. The result is a highly efficient, though sub-optimal, outcome and a generous treatment of poor countries. The cost of such a policy, even for the United States, is only a few dollars per person per year.

The next section introduces notation, and the following sections solve the various games. This paper is primarily designed to serve as technical background paper for a subsequent policy paper, but by skipping the proofs and reading the short text in between, the reader will gain easy access to the results.

2 Framework and Notation

This paper analyzes climate policies by computing Nash equilibria. This requires defining games. Each game consists of a model global economy and a climate policy. Economies and policies are paired in various ways and are fairly independent components, so they will be defined separately in advance and then linked to form the games as needed. Model economies and policies share some common notation summarized below.

2.1 Notation

Table 1. Common Notation

$N, \ \Sigma$	N = the number of countries. Sums are over all countries	(cn1)
$A = \Sigma A_j$	Global total abatement = the sum of national abatements	(cn2)
$T = \Sigma T_j$	Global abatement target = sum of national targets	(cn3)
$B = \Sigma b_j$	Total benefit coefficient = sum of national coefficients	(cn4)
$B_j(A)$	Benefit function, usually equal to $b_j A$	(cn5)
$C(A_j) = c_j A_j^2$	Cost of abatement assumed to be quadratic	(cn6)
$k = \Sigma (1/c_j)$	Sum of reciprocal cost-of-abatement coefficients	(cn7)
$GNB = \Sigma NB_j$	Global net benefit = the sum of national net benefits	(cn8)
$P = P_j, \ P_j = 2 c_j A_j$	The global carbon price, $P = P_j$ for all *j* when there is a global carbon market. Country j's carbon price = marginal cost	(cn9)
$E = \Sigma E_j$	Global emissions = sum of national emissions	(cn10)
$n = \Sigma n_j$	Global population = sum of national populations	(cn11)
$e = E/n, \ e_j = E_j / n_j$	Global and national emissions per capita	(cn12)
$b_j = b E_j$	Benefit coefficients in standard public goods (PG) economies	(cn13)
$c_j = c / E_j$	Cost coefficients in standard PG economies. Scaled for size.	(cn14)
$g_j P$, where $g_j = g(e_j - e) n_j$	Green-Fund payment from country j	(cn15)
$c_j b_j = cb, \ cbk = B$	Identities based on (cn4), (cn13), and (cn14)	(cn16)

2.2 Models of the Global Economy

Two fundamental global economies are considered—public-goods economies which exhibit the free-rider problem, and rich-and-poor economies which do not. Economies that combine both fundamental attributes will also be considered. In the "standard pubic-goods model" countries differ only by size. This is sufficient to exhibit the public-goods problem that is the fundamental problem of climate policy. In fact, even when all are the same size the problem is on display, but size variation plays an interesting role and one that is crucial for understanding cap and trade. For comparison, cooperative economies, which exhibit no public-goods problem will also be analyzed.

Table 2. Model Economies

PG: The Standard Public-Goods Economy		
$\mathrm{NB_j} = b_j A - c_j A_j^2$	Net benefit of country j depends on global abatement	(me1)
$b_j = b E_j,\ \ c_j = c/E_j$	Benefits and costs scale with size of national economy	(me2)
The Arbitrary Public-Goods Economy		
$\mathrm{NB_j} = b_j A - c_j A_j^2$	Net benefit depends on global abatement	(me3)
$b_j,\ c_j$	Cost and benefits not related to size of economy	(me4)
The Cooperative Rich-and-poor Economy		
$\mathrm{NB_j} = B A - (e/e_j)\, c_j A_j^2$	Poor (low emission) countries see higher-than-market costs. Countries consider global benefits when choosing their abatement level.	(me5)
RP#1: The Public-Goods, Rich-and-Poor Economy #1		
$\mathrm{NB_j} = b_j A - (e/e_j)\, c_j A_j^2$	Poor (low emission) countries see higher abatement costs	(me6)
$b_j = b E_j,\ \ c_j = c/E_j$	Benefits and costs scale with size of national economy	(me7)
RP#2: The Public-Goods, Rich-and-Poor Economy #2		
$\mathrm{NB_j} = b_j A - (e/e_j)\,[\, c_j A_j^2 - \mathrm{Income}\,]$	Low-e_j countries see all monetary transactions magnified	(me8)
$b_j = b E_j,\ \ c_j = c/E_j$	Benefits and costs scale with size of national economy	(me9)

Various policy mechanisms will be added to these model economies to produce different games with which to test the mechanism. These policies are now defined.

2.3 Policy Mechanisms

Table 3. Policy Mechanisms

The No-Policy Mechanism		
$d\mathrm{NB_j} / dA_j = 0$	Countries independently optimize abatement	(pm1)
CT: The Cap-and-Trade Mechanism		
$d\mathrm{NB_j} / dT_j = 0$	Countries choose their target to maximize their net benefit	(pm2)
$P = 2\, c_j A_j$	The global carbon price, the marginal cost of abatement, clears all markets.	(pm3)
The Global-Price-Target Mechanism		
$A = A(P_j^*),\ \ d\mathrm{NB_j} / dP_j^* = 0$	Desired price-targets optimized, assuming P_j^* will affect abatement in all countries.	(pm4)
$P = \min(P_j^*)$	The minimum P_j^* determines the global price target	(pm5)
The Price-Target-with-Green-Fund Mechanism		
$g_j P$ is subtracted from $\mathrm{NB_j}$	Same as Global-Price-Target but with a Green-Fund paid for by high-emission countries and paid to low-emission countries.	(pm6)

3 The Public-Goods/No-Policy Game

The public-goods/no-policy game is perhaps the most familiar game of those considered here, so we solve it first. Each country maximizes its own net benefit with no consideration for others. This gives a Nash equilibrium and the Nash strategies are all strictly dominant—they are the best strategy for each country no matter what strategy the other countries pursue. As is well know this outcome is suboptimal. (This analysis holds for a one-shot game, but not necessarily for an infinitely repeated game.)

Table 4. Solution of the Arbitrary Public-Goods/No-Policy Game

$\text{NB}_j = b_j A - c_j A_j^2$	(me3) Net benefit to country j	(a1)
$d\text{NB}_j / dA_j = 0$	(pm1) Countries independently optimize abatement	(a2)
$dA/dA_j = 1$	(cn2) Differentiate	(a3)
$b_j - 2 c_j A_j = 0$	(a2) (a3)	(a4)
$A_j = b_j / (2 c_j)$	(a4) Nash abatement strategy	(a5)
$P_j = 2 c_j A_j$	(cn9) Country j's price = marginal cost	(a6)
$P_j = b_j$	(a5) (a6) Equivalent Nash pricing strategy	(a7)

Note that this has the same solution as the *standard* public-goods/no-policy game. Since the public goods game leads to a suboptimal outcome, it is useful to find the optimal outcome to use as a benchmark. This can be done by having countries optimize the global net benefit instead of their own net benefit.

Table 5. Solution of the Public-Goods/Cooperation Game

$\text{NB}_j = b_j A - c_j A_j^2$	(me3) Net benefit to country j	(b1)
$\text{GNB}_j = (\Sigma\, b_j)\, A - c_j A_j^2 -$ costs in other countries	(cn8) (b1)	(b2)
$d\text{GNB} / dA_j = 0$	(pm2) Countries optimize for global net benefit	(b3)
$\Sigma\, b_j - 2 c_j A_j = 0$	(b2) (b3)	(b4)
$A_j = (\Sigma\, b_j) / (2 c_j)$	Globally optimal A_j	(b5)
$P_j = 2 c_j A_j$	Define price in j to be its marginal cost of abatement	(b6)
$P_j = \Sigma\, b_j = B$	(b5), (b6) and (cn4)	(b7)

4 The Standard Public-Goods/Cap-and-Trade Game

The PG-CT Game adds to the PG Game a cap-and-trade market in each country and links these markets with international trading to produce the global "carbon market." The game then consists of each country choosing a cap, or abatement target, instead of a national abatement level. The international carbon market then determines the national abatement levels.

The existence of the market raises the question of cheating and enforcement. Because the caps are chosen voluntarily as Nash strategies, there is no reason for countries to cheat and implement a different cap than they choose for their strategy. (A Nash strategy is one that's best given others' actual strategies, so no information advantage can be gained by cheating.) However a country could benefit by letting its industries emit without retiring the emission permits that enforce the cap. These could then be sold for a profit on the international market. This would save the country the cost of abatement. If such cheating is possible, then no one would abate. The rules of the present game assume that countries do not allow such cheating and that domestic cap-and-trade markets are perfectly enforced.

As is customary, the caps will be described by how much they will cause a country to reduce its emissions below business as usual. In other words, caps are implemented as abatement targets, T_j. According to the game's rules, a target can be complied with by domestic abatement or by purchasing abatement in the international carbon market.

With countries free to choose any target, even a negative target that allows them to issue more emission permits than their business-as-usual emissions, it might be thought that a no-abatement Nash equilibrium would be dominant, just as it is with cheating. If so, this would still be of interest. It would tell us that without a central authority capable of dictating (or at least limiting) the choice of national targets, the introduction and acceptance of a perfectly functioning global cap-and-trade mechanism would result in the complete collapse of the limited cooperation that occurs in the PG Game.

This does not occur. Free choice of targets does not put an end to the voluntary supply of abatement, the public good.

With identical countries, the free choice of targets (caps) turns out to be equivalent to the free choice of abatement levels in the PG game. In this case, the two games generate identical behavior. Only as the world becomes asymmetric does the CT Game's outcome gradually diverge from GP Game's outcome. The equivalence of PG and CT Games with identical countries will be demonstrated first.

Although not discussed in this paper, the CT Game should be compared with the PG Game when considering a global agreement which pressures countries into accepting tighter caps. If the CT Game is inherently less cooperative than the PG Game, then the agreement will need to apply more pressure to achieve the same level of cooperation as could be achieved without cap and trade.

Table 6. Rules of the Public-Goods/Cap-and-Trade (CT) Game

The CT Game includes all the rules of the PG Game except for the definition of net benefit, which is changed.

$T = \Sigma\, T_j$	(cn3) Total global abatement target	(c1)
$A = T$	Compliance with the targets (caps)	(c2)
$P = 2\, c_j A_j$	(pm3) The global carbon price, the marginal cost of abatement, clears all markets	(c3)
$\text{NB}_j = b_j A - c_j A_j^2 - P\cdot(T_j - A_j)$	The net benefit function also embodies the compliance assumption	(c4)
$d\text{NB}_j \,/\, dT_j = 0$	(pm3) Countries choose their target to maximize their net benefit	(c5)

The cap is imposed by issuing carbon permits which must be retired to cover emissions, and any left-over permits can be sold nationally or internationally. The carbon trading term in the net benefit definition can be checked as follows. Suppose a country abates by A_j but is paid by other countries for xj of exported emission permits and has target T_j. We need to know its cost of ***net*** imported emission permits. Compliance with the cap and trade regulations allow us to find this amount. Only $A_j -$ xj counts toward the target, T_j, and so $T_j - (A_j -$ xj) emission permits must be imported (purchased from other countries). Net imports are then $T_j - (A_j -$ xj) $-$ xj, or simply $T_j - A_j$, with a net cost of $P_j \cdot (T_j - A_j)$. Hence the net benefit formula embodies compliance and correctly accounts for the international trade of carbon permits.

This is a two stage game, and so, as is typical, we begin by solving the second stage—the carbon market. We then move on to solve the first stage, finding the Nash Equilibrium choice of targets. As noted, the symmetric version of the CT Game will be solved first in order to demonstrate that the CT Game is reasonably constructed and to show the important result that it has the same solution as the PG Game in a symmetric world.

4.1 The Symmetric Cap-and-Trade Game

In this game, all N countries are the same size and identical in all respects. However, they act with complete independence. Instead of choosing abatement, countries choose the number of internationally tradable carbon permits they will issue. Surprisingly there is no gaming of the opportunity to profit from printing valuable permits, and the outcome is the same as without cap and trade. This result appears to be non-trivial.

Table 7. Solving the Symmetric Public-Goods/Cap-and-Trade Game

Defining the symmetric CT Game		
$\mathrm{C}(A_j) = c\,A_j^2,\ \ \mathrm{B}_j(A) = b\,A$	All countries share the same coefficients, c and b. [These are different from c & b defined in (cn13 & cn14)]	(d1)
$A = N{\cdot}A_j\,,\ \ T = N{\cdot}T_j$	There are N identical countries	(d2)
Solving Stage 2, the Carbon Market		
$P = 2\,c\,A_j$	(cn9) Price equals marginal cost in all countries	(e1)
$A_j = A/N$	(d2)	(e2)
$P = 2\,c\,A/N$	(e1) and (e2)	(e3)
Solving Stage 1: Finding the Nash-Equilibrium Targets		
$A' = dA\,/\,dT_j = dT/dT_j = 1$	Defines A' and (c1) and (c2).	(f1)
$P' = dP/dT_j = 2c/N$	Defines P' and (e3) and (f1).	(f2)
$A_j' = dA_j/dT_j$	Defines A_j'	(f3)
$\mathrm{NB}_j = b\,A - c\,A_j^2 - P{\cdot}(T_j - A_j)$	(c4)	(f4)
$d\mathrm{NB}_j\,/\,dT_j = 0$	First order condition	(f5)
$b - 2\,c\,A_j\,A_j' - P'(T_j - A_j) - P{\cdot}(1 - A_j') = 0$	Differentiate, (f1)	(f6)
$b - 2c{\cdot}(A/N)\,A_j' - P'{\cdot}(T_j - A_j) - 2c{\cdot}(A/N)(1 - A_j') = 0$	(e2) and (e3)	(f7)
$b - P'(T_j - A_j) - 2cA/N = 0$	Cancel the $2c{\cdot}(A_j/N)A_j'$ terms	(f8)
$b - (2c/N)(T_j - A_j) - 2cA/N = 0$	(f2)	(f9)
$(2c/N)\,T_j = b + (2c/N)A_j - (2c/N)A$		(f10)
$T_j = b/(2c/N) + A/N - A$	(e2)	(f11)
$\Sigma\ T_j = \Sigma\,[N\,b/(2c)] + T{\cdot}(1 - N)$	Sum over N countries, and (c2)	(f12)
$T = N^2 b\,/2c + T{\cdot}(1 - N)$	(c1) and (c2)	(f13)
$T = N\,b\,/2c$	Solve for T	(f14)
$T_j = b\,/2c$	(d2) This is the Nash equilibrium and it is the same as (a5) for a symmetric PG Game.	(f15)

The only difference between the symmetric CT Game and the symmetric PG Game is that in one, countries choose abatement levels and in the other, that choose abatement targets. But, while they are free to fulfill their targets by purchasing abatement elsewhere, since all countries are identical the abatements match the targets country by country. There is simply no real difference between these two games.

This result is important because it shows that while countries are free to choose a negative abatement target (a cap above business as usual) and use the international carbon credits generated by such a target to profit from the targets of other countries, they freely choose not to do so.

Result 1: The CT Game does not succumb to low or negative targets simply because countries are free to profit from setting such targets. In fact, when countries are identical, they exhibit no tendency in this direction, but choose the PG Game solution precisely.

4.2 The Asymmetric Cap-and-Trade Game

The approach to solving the asymmetric CT Game is essentially the same, but the outcome no longer mimics the outcome of the PG Game. Instead the strategies of countries become more polarized and total abatement often falls, as will be explained in three theorems. Again these results cannot be explained by a simple theory of profiting by printing international carbon credits as if they were free money—even though there is no rule against that.

Table 8. Solving the Arbitrary Public-Goods/Cap-and-Trade Game

Solving Stage 2, the Carbon Market		
$P = 2\, c_j A_j$ for all j	(pm3)	(g1)
$A_j = (P/2)(1/c_j)$	(g1)	(g2)
$A = \Sigma A_j = (P/2)\,\Sigma (1/c_j)$	(cn2) and (g2)	(g3)
$k = \Sigma (1/c_j)$	(cn6) defines k	(g4)
$T = (P/2)\, k$	(g3), (g4) and (c2)	(g5)
$P = 2\, T/k$	(g5)	(g6)
$A_j = T/(c_j k)$	(g2) and (g6)	(g7)
Solving Stage 1: Finding the Nash-Equilibrium Targets		
$A' = dA/dT_j = dT/dT_j = 1$	Definition of A', (c2) and (c1).	(h1)
$P' = dP/dT_j = 2/k$	Definition of P', (g6) and (h1)	(h2)
$A_j' = dA_j/dT_j$	Definition of A_j'	(h3)
$NB_j = b_j A - c_j A_j^2 - P\cdot(T_j - A_j)$	(c4)	(h4)
$dNB_j / dT_j = 0$	First order condition	(h5)
$b_j - 2\, c_j A_j A_j' - P'(T_j - A_j) - P\cdot(1 - A_j') = 0$	Differentiate	(h6)
$b_j - P A_j' - P'(T_j - A_j) - P\cdot(1 - A_j') = 0$	(g1)	(h7)
$b_j - P'(T_j - A_j) - P = 0$	(h7) and cancel $P A_j'$	(h8)
$b_j - (2/k)(T_j - A_j) - 2T/k = 0$	(h2) and (g6)	(h9)
$T_j = (k/2) b_j + A_j - T$	Solve for T_j	(h10)
$T_j = (k/2) b_j + T/(c_j k) - T$	(g7)	(h11)
$\Sigma T_j = (k/2)\, \Sigma b_j + T\cdot(\, [\Sigma (1/c_j)]/k) - N)$	Sum (h11) over j	(h12)
$T = (k/2)\, \Sigma b_j + T\cdot(1 - N)$	(g4)	(h13)
$N T = (k/2)\, \Sigma (b_j)$	Solve for T	(h14)
$T = [\Sigma (1/c_j)]\cdot[\Sigma b_j] / 2N = k B / 2N$	(g4), (cn4)	(h15)
$T_j = k b_j / 2 + T/(c_j k) - T$	(h11) Nash equilibrium	(h16)
$A_j = (B/N)/(2\, c_j)$	(g7) Abatement outcome	(h17)

Note that the result is consistent with the result for the symmetric case. If c_j = c and b_j = b, then (h15) simplifies to [N/c][Nb]/2N = Nb/2c, which is the same as (f14). Numerical calculations along with the solution to stage-2 (the carbon market) can be used to check the more difficult solution to stage 1. Table 9 presents the solution of a two-country game in which country j=1 is twice as large as country j=2. Consequently country 1 receives twice the benefit and can abate the same quantity (half the percentage abatement) at half the cost.

Table 9. Outcome of a two-country cap-and-trade game

	Input				Solution			
j	***N***	***c_j***	**$1/c_j$**	***b_j***	***T***	***T_j***	***A_j***	***P***
1	2	1	1	2	1.125	1.125	0.75	1.5
2		2	0.5	1		0	0.375	
			1.5	3		1.125	1.125	
			k			***T***	***A***	

For comparison, the PG Game equilibrium is $A_1 = 1$, $A_2 = 0.25$. Hence, we see that in the CT Game, the country that abates most per capita in the PG Game, targets (T_j) even more abatement in the CT Game, while the country that free-rides most (per capita) in the PG Game, free-rides even more in the CT Game. Also, the total abatement declines from 1.25 to 1.125. So adding a cap-and-trade mechanism causes a polarization of the players and reduces overall efficiency. However what is abated is abated more efficiently because the carbon market equalizes marginal costs.

The next step is to check whether the values of T_j, 1.125 and 0, actually optimize the net benefit of the two countries (given the other country's target). This will be done numerically as a way of showing that the calculations for stage 1 are correct.

Table 10. Checking that the analytically computed Nash equilibrium is correct

					Use Stage-2 Calculations			Check	
j	***c_j***	***b_j***	**ΔT_j**	**Test T_j**	***T***	***A_j***	***P***	NB_j	
1	1	2	-0.1	1.03	1.03	0.68	1.37	1.12	
1	1	2	**0**	1.13	1.13	0.75	1.5	1.13	max
1	1	2	0.1	1.23	1.23	0.82	1.63	1.12	
2	2	1	-0.1	-0.1	1.03	0.34	1.37	1.40	
2	2	1	**0**	0	1.13	0.38	1.5	1.41	max
2	2	1	0.1	0.1	1.23	0.41	1.63	1.40	

The check is performed by computing the outcome of targets that deviate slightly from the computed Nash-equilibrium target. The deviations are listed under ΔT_j and are plus and minus 0.1. In both cases, both deviations cause a decrease in the net benefit, NB_j. The calculation of NB_j uses the following formulas

Table 11. Checking the Stage 1 Calculations Numerically

Formulas used to compute net benefit from the Nash equilibrium predicted by the Stage 1 calculations

$T = T + dT_j$	Global target is change along with T_j	(i1)
$A_j = T / (c_j\ k)$	(g7)	(i2)
$P = 2\ c_j A_j$	(g1)	(i3)

The first formula (i1) follows from the Nash condition which specifies a change in one strategy with all others held constant. Since the others are held constant, the global target is increases by dT_j.

The second equation (i2) gives country j its proportional share of the increased abatement.

Finally, formula (i3) gives the price that results with the extra targeted abatement, ΔT_j, being divided economically among all countries. Together these three variables, and the input parameters determine the net benefit. This determination depends only on the Nash targets that have been selected and the efficient operation of the carbon market. Table 10 shows that if either of the strategies (targets) deviate from their computed levels, the country's net benefit decreases. Hence we have a Nash equilibrium. This checks one application of the stage-1 analysis. Many others have been checked the same way, so it is highly likely that the stage-1 analysis is correct. The stage-2 market analysis is much simpler, so it is unlikely to be in error.

4.3 The CT-Game Price Theorem

The CT-Game Price Theorem: Assume abatement costs are quadratic and the benefit of the public good is linear, as described above. then the market price under cap and trade will be $P = (\Sigma\, b_j) \,/\, N$, where N is the number of countries.[1]

Note that the optimal price is $P = (\Sigma\, b_j)$, so the cap and trade price is N times too low. The proof is to substitute the formula for T from (h15) into (g6). This theorem shows that while the public-goods game exhibits better outcomes when some countries are much larger than others, the CT Game does not. It's outcome is independent of the distribution of country sizes.

4.4 The CT-Game Polarization Theorem

In the two country example, we see that the country that abates less per-capita in the PG Game chooses a cap-and-trade target that is even lower than its PG-Game abatement level. Conversely, the country that is more cooperative in the PG Game, becomes yet more cooperative in the CT Game. This behavior points to an important intuition concerning the outcome of cap and trade.

In the PG Game, countries supply abatement up to the point at which its marginal cost equals its marginal benefit. Call that value P_j. Cap-and-trade makes it possible for countries that have chosen to abate up to a higher price level to, instead, abatement at a lower cost by buying abatement from countries that have chosen little abatement. The result is a uniform marginal cost of abatement across all countries that is between the lowest and highest values of P_j in the PG Game. Call this uniform value P. Countries that find $P > P_j$ will find their marginal cost of abatement has increased, and will wish to do less, while those that find $P < P_j$, will wish to abate more than they did in the PG Game.

This insight is formalized in the following cap-and-trade polarization theorem. To state the theorem, define a low-abatement country as one with lower abatement in the public-goods (PG) game than its abatement level (not its target level) in the cap-and-trade (CT) game. A high-abatement country is defined analogously. Also, define a country's emission target in the PG Game as its abatement level.

The CT-Game Polarization Theorem: Adding the cap-and-trade mechanism to the public-goods game causes low-abatement countries to lower their emission targets and high-abatement countries to raise their emission targets.

Proof: The proof for high-abatement countries is the mirror image of that for low-abatement countries, so only the low-abatement proof will be given. Low-abatement countries have A_j [PG] < A_j [CT], which means they also have P_j [PG] < P [CT], where the bracketed abbreviation indicates which game the variable is from. [It is important to remember that A_j [PG] < A_j [CT] does not mean that the country is responsible for more abatement in the CT game. It only means others are paying it to increase its abatement.] We wish to show that T_j [CT] < A_j [PG].

NB_j will refer to the CT Game unless followed by [PG]. P_j will always mean a country's marginal cost of abatement in the PG game, since there is only one price, P, in the CT Game.

Table 12. Proof of Polarization Theorem:

Assume that in CT equilibrium: $P_j < P$ but $T_j > A_j$[PG]	This is a proof by contradiction	(j1)
$NB_j' = b_j - P'(T_j - A_j[CT]) - P = 0$	From (h8) above. Note: prime (') means $d(\,)/dT_j$.	(j2)
$NB_j'[PG] = b_j - P_j = 0$	From (a4) with Pj = 2 cj Aj[PG]	(j3)
$NB_j' = -P'(T_j - A_j[CT]) - P + P_j$	Subtract zero (j3) from (j2)	(j4)
$NB_j' = -P'(T_j - A_j[CT]) - (2\, c_j\, A_j[CT] - 2\, c_j\, A_j[PG])$	Replace price with marginal cost	(j5)

[1] It is conjectured that the assumptions concerning the cost and benefit functions are much stronger than necessary.

$\Delta Aj \; = \; A_j\,[CT] \; - \; A_j\,[PG]$	Definition	(j6)
$A_j\,[PG] \; < \; A_j\,[CT]$	Because $P_j\,[PG] < P\,[CT]$, (j1)	(j7)
$\Delta Aj \; > \; 0$	From (j7)	(j8)
$NB_j' \; = \; P'(A_j\,[CT] \; - \; T_j) \; - \; 2\; c_j\; \Delta Aj$	From (j5) and (j6)	(j9)
$A_j\,[CT] \; - \; T_j \; < \; A_j\,[CT] \; - \; A_j\,[PG]$	From (j1)	(j10)
$NB_j' \; < \; P'\cdot(A_j\,[CT] \; - \; A_j\,[PG]) \; - \; 2\; c_j\; \Delta Aj$	(j9) and (j10)	(j11)
$NB_j' \; < \; 2\; c_j\; A_j'\cdot(\; \Delta Ai) \; - \; 2\; c_j\; \Delta Aj$	(j6), differentiate (c3) to get $P' \; = \; 2\; c_j\; A_j'$	(j12)
$NB_j' \; < \; 2\; c_j\; \Delta Aj\; (A_j' - 1)$	(j12)	(j13)
$Aj' < 1$	Because A_j is the fraction of T that goes to country j, so when T_j increases by 1, T increases by 1, and A_j increases by less than 1.	(j14)
$NB_j' \; < \; 2\; c_j\; \Delta Aj\; (A_j' - 1) \; < \; 0$	(j8), (j13) and (j14)	(j15)

The final step proves the system is not at an Nash equilibrium, otherwise NPj' would be zero. This contradicts (j1) so the theorem is true. Also note that since NPj' < 0, country j would reduce T_j from the assumed value which moves it towards $T_j\,[CT] < A_j\,[PG]$ as claimed by the theorem.

4.5 The CT-Game Total Abatement Theorem:

In the example above, adding a cap and trade mechanism to the PG Game reduces the level of global abatement. The Total Abatement Theorem says this occurs in a significant family of CT Games.

The CT-Game Total Abatement Theorem: Assume abatement costs are quadratic and the benefit of the public good is linear, as described above. If b_j is positively correlated with $1/c_j$ then adding a global cap-and-trade mechanism to the no-policy, public goods game, as described above, will decrease total abatement.

Proof:

Definitions: X and Y are vectors with components {X1 … Xn} and {Y1 … Yn}. X·Y is the Euclidian inner product—the standard "dot" product, ∑ Xj Yj. The mean of X is mx, and of Y is my. The vector Mx has n identical components: {mx … mx}, and My = {my … my}. $\mathbf{X}$ = X – Mx. $\mathbf{Y}$ = Y – My.

Table 13. Proof of Total Abatement Theorem

$Xj \; = \; 1/cj, \;\; Yj \; = \; bj$	Definitions	(k1)
$Cor(1/cj,\, bj) > 0$ implies	The "If" part of the theorem	(k2)
$Cor(X,Y) > 0$ implies		(k3)
$\mathbf{X}\cdot\mathbf{Y} > 0$ implies $N\,\mathbf{X}\cdot\mathbf{Y} \; > \; 0$	From definition of sample correlation	(k4)
Implies $N\,\mathbf{X}\cdot\mathbf{Y} \; > \; \Sigma\,\mathbf{X}\,\Sigma\,\mathbf{Y}$	Because $\Sigma\,\mathbf{X} = \mathbf{0}$	(k5)
$N\,\mathbf{X}\cdot\mathbf{Y} + N\,my\,\Sigma\mathbf{X} + N\,mx\,\Sigma\mathbf{Y} + N^2\,mx\,my \;\; > \;\; \Sigma\,\mathbf{X}\,\Sigma\,\mathbf{Y} + N\,my\,\Sigma\mathbf{X} + N\,mx\,\Sigma\mathbf{Y} + N^2\,mx\,my$		(k6)
$N\;\; [\,\mathbf{X}\cdot\mathbf{Y} + \mathbf{X}\cdot My + \mathbf{Y}\cdot Mx + Mx\cdot My\,] \;\; > \;\; \Sigma\,\mathbf{X}\,\Sigma\,\mathbf{Y} + \Sigma\,\mathbf{X}\,\Sigma\,My + \Sigma\,Mx\,\Sigma\,\mathbf{Y} + \Sigma\,Mx\,\Sigma\,My$		(k7)
$N\,[\,(\mathbf{X} + Mx)\cdot(\mathbf{Y} + My)\,] \;\; > \;\; [\,\Sigma\,(\mathbf{X} + Mx)j\,] \times [\,\Sigma\,(\mathbf{Y} + My)j\,]$, sum from j=1 to n		
$N\,\Sigma\,[(\mathbf{X} + Mx)j \times (\mathbf{Y} + My)j] \;\; > \;\; [\,\Sigma\,(\mathbf{X} + Mx)j\,] \times [\,\Sigma\,(\mathbf{Y} + My)j\,]$		(k8)
$\Sigma(Xj\,Yj) \; > \; [\,\Sigma\,Xj\,]\,[\,\Sigma\,Yj\,]\,/\,N$		(k9)
$\Sigma(b_j\,/\,c_j) \; > [\,\Sigma(1/c_j)\,]\,[\,\Sigma(b_j)\,]\,/\,N$	From (k1)	(k10)

$$[\,\Sigma(b_j/c_j)]\,/\,2 \;>\; [\,\Sigma(1/c_j)\,]\,[\,\Sigma(b_j)\,]\,/\,2N \qquad \text{(k11)}$$

$$A_{PG} \;>\; T_{CT} \quad \text{From (a5) summed, and (h15)} \qquad \text{(k12)}$$

Table 13 shows when abatement, A, in the public-goods game, is greater than the global target, T, in the cap-and-trade game. In the *standard* public goods game, in which countries differ only buy size, the required correlation is sure to hold. In any arbitrary public goods game, unless there is some reason for a negative correlation between $1/c_j$ and b_j it will almost certainly hold. So it seem very likely that adding a cap-and-trade mechanism will reduce total global abatement.

5 Rich-and-Poor Games

Finding a cooperative solution to the climate game is not just a matter of solving the free rider problem. Other forces also make cooperation difficult. In particular there has been a schism between rich and poor countries. The free-rider problem varies by size, and income is not well correlated with size, which shows that the rich-and-poor divide is not caused by the free-rider problem. To round out the critique of cap-and-trade and to test alternative policy mechanisms, we will need a model of how the rich-and-poor differential is connected to cooperation. To that end we introduce rich-and-poor model economies.

Countries may commit to supply different levels of a public good for reasons other than having different free-rider incentives. Some of these reasons have to do with income per capita, which we will model as being linked to emissions per capita. The reasons for differing commitments may affect a country's net-benefit function in a number of ways, but one of the easiest effects to analyze is the following:

$$NB_j \;=\; B_j(A) - (e/e_j)\,C(A_j), \qquad \text{(m1)}$$

where e is the global-average emissions per capita and e_j is emission per capita for country j. Emissions per capita is used as a proxy for income both for convenience and because basing the Green Fund on e_j provides excellent incentives, while basing it on income does not. In any case, this models a world in which poor countries view the cost of abatement as relatively more significant compared to climate benefits, than do rich countries.

To make the distinction between the public-goods problem and the rich-and-poor problem as clear as possible, we begin by analyzing an economy that has no public-goods problem—a cooperative economy. This is not a realistic model, but it is useful for clarifying the theory.

Table 14. Comparing the Cooperative Rich-and-poor Economy to the Public-Goods Economy

$NB_j \;=\; (\,\Sigma\, b_j)\,A \;-\; (e/e_j)\;c_j\;A_j^2$	Net benefits of countries in a cooperative rich-and-poor economy with no policy.	(m2)
$d\,NB_j\;/\;dA_j \;=\; 0$	First-order optimization conditions	(m3)
$d\,[(e_j/e)NB_j\,]/\;dA_j \;=\; 0$	Equivalent set of conditions	(m4)
$h_j \;=\; (e_j/e)\;B$	Definition of h_j ($B = \Sigma\, b_j$)	(m5)
$NBh_j \;=\; (e_j/e)NB_j \;=\; h_j\,A - c_j\,A_j^2$	Definition of NBh_j. (m2) (cn4)	(m6)
$d\,NBh_j\;/\;dA_j \;=\; 0$	Equivalent first-order optimization conditions	(m7)

This shows that the cooperative rich-and-poor/no-policy game has the same solution as the arbitrary public-goods/no-policy game (a1) with $b_j = h_j$.

Now consider the cooperative rich-and-poor/cap-and-trade game.

$$\text{NB}_\text{j} = \Sigma\, b_j A - (e/e_j)\,[\,c_j\, A_\text{j}^2 - P\cdot(T_j - A_\text{j})\,]$$

Net benefits of countries in a cooperative rich-and-poor economy with a cap-and-trade policy. (m8)

$$\text{NB}_\text{j} = h_j A - c_j\, A_\text{j}^2 - P\cdot(T_j - A_\text{j})$$

Definition of net benefits that would give an equivalent first order conditions. (m9)

This shows that the cooperative rich-and-poor/cap-and-trade game has the same solution as the arbitrary public-goods/cap-and-trade game with $b_j = h_j$.

Table 14 demonstrates that a cooperative rich-and-poor economy (as defined above) has the same outcomes as a particular (non-standard) public goods economy under either no policy or under a cap and trade policy. Consequently the three theorems that relate no-policy to cap-and-trade for a public-goods economy also relate no-policy to cap-and-trade in a rich-and-poor economy.

But the theorems tell us the relationship between no-policy and cap-and-trade depends on the coefficients that describe the economy. Hence it is important to remember that a rich-and-poor economy with { b_j, c_j, and e_j, } corresponds (by Table 14) to a public goods economy with { h_j , c_j }, where $h_j = (e_j\,/\,\text{e})\ \Sigma\ b_j$.

The price theorem now says the cooperative rich-and-poor/cap-and-trade price will be $(\Sigma\ h_j)/\text{N}$. But this is not such a bad result as before. In the symmetric public goods game $\Sigma\ b_j = B$, so the cap-and-trade price was B/N, which is N times too low. But, in the symmetric rich-and-poor game, $\Sigma\ h_j = N\,B$, so the rich-and-poor cap-and-trade price would be exactly right.

Similarly, the rich-and-poor/cap-and-trade game may not suffer from low total abatement because that result holds only when b_j and $1/c_j$ are correlated, and with b_j replaced by h_j, there is no reason to think this condition holds.

However, the polarization theorem will still hold because it does not depend on the properties of the cost and benefit coefficients. This is best understood by returning to the intuitive explanation of that theorem. In the no-policy game, some country will choose a low marginal cost of abatement and others will choose a higher marginal cost for their abatement strategy. This is true in both the public-goods and the rich-and-poor economies. The global abatement market under a cap-and-trade policy equalizes these marginal costs at an intermediate level.[2] Countries that desire to abate only up to a lower marginal cost will find the new global price of abatement too high and choose to do less, while countries that find the new global price lower than what they are willing to pay will choose to do more. So again cap-and-trade causes polarization.

6 Price Targets and Green Funds

We have found that when a cap-and-trade mechanism is added to the climate game it causes polarization and most likely leads to a decrease in emission abatement. This is not surprising. Cap-and-trade was selected by historical accident with no thought for free-rider problems or how to solve them. Next we turn to a pair of mechanisms designed to work together to encourage cooperation.

A global price target applies to all countries equally and is selected by all countries together through a voting procedure. The full game works as follows:

1. A Green-Fund formula that depends on the agreed price target is selected. In the pure price-target case, the formula sets all payments to zero.
2. Countries vote for any price target they like.
3. The lowest price target voted for is selected.
4. Countries post bonds somewhat greater than the cost of full compliance for one year.
5. If all countries post bonds, then all countries must price carbon at the global price target and make payments (if any) according to the Green-Fund formula.

[2] Though it has been observed in many games, this still needs to be proven.

6. If any country fails to comply, it forfeits its bond.
7. The process is repeated annually

These rules would need modification for use in the real world. For example the Kyoto treaty was set to take affect with only something like 55% of emissions covered by signers. The above specification is designed to show that an efficiently-cooperative, self-enforcing treaty is possible in theory.

To see how the game can have a fully cooperative Nash equilibrium, analyze the game backward. With all countries having voted for the optimal price and posted their bonds, it makes sense for all of them to implement the optimal price. If one does not, it will lose its bond and save no money. Plus, the climate will be a bit worse.

With all countries having voted for the optimal price, they may as well post their bond, since this has no effect unless all others do likewise, and in that case, we just saw that the outcome will be optimal. While if they don't post their bond, no one will take action and all will be worse off.

We assume without proof that if a country's preferred prices target is T*, then any lower price target would be better for them than no price target. In this case, they should vote for their preferred price target. Now the only remaining step is to find the ideal target for each country and show that selecting the lowest one from this set produces a high level of cooperation.

6.1 The Public-Goods Game with a Price Target

With the standard public goods game, no Green Fund is needed and a global price target completely overcomes the free rider problem by, in effect, internalizing the externality. This game is solved first.

Table 15. Solution of the Standard Public-Goods/Price-Target Game

$\mathrm{NB_j} = b_j A - c_j A_j^2$	(me1)	(p1)
$bj = b\, E_j$	(me2)	(p2)
$cj = c\,/\,E_j$	(me2)	(p3)
$P = 2\, c_j A_j$	(cn9)	(p4)
$A_j = P\,/\,(2\, c_j)$	(p4)	(p5)
$A = \Sigma\, A_j = (P/2)\, \Sigma\, (\, 1/c_j\,)$	(cn2), (p5)	(p6)
$\mathrm{NB_j} = (b_j P/2)\, \Sigma\, (\, 1/c_j\,) - c_j \cdot (P\,/\,2\, c_j)^2$	(p1), (p6)	(p7)
$\mathrm{d\, NB_j} \,/\, \mathrm{dP} = 0$	(pm2)	(p8)
$0 = (b_j/2)\, \Sigma\, (\, 1/c_j\,)\,) - P/(2\, c_j)$	(p7), (p8)	(p9)
$P_j = c_j\, b_j\, \Sigma\, (\, 1/c_j\,)$	(p9)	(p10)
$P_j = (c/E_j)\, (b\, E_j)\; \Sigma\, (\, E_j/c\,)$	(cn13), (cn14)	(p11)
$P_j = c\; b\; (1/\mathrm{c})\, \Sigma\, E_j$	(p11)	(p12)
$P_j = b\, \Sigma\, E_j = B$	(cn4), (cn13) Which is the optimal price	(p13)
$A_j = B\; /\; 2 c_j$	Which is the optimal solution according to (b5)	(p14)

In the same idealized world in which cap-and-trade induces a global carbon price that is N times too low, the global-price-target mechanism induces the optimal carbon price. When each country optimizes price, it understands that raising the price will make all other countries provide more of the public good to its benefit. This is not the same as internalizing the positive externality of better climate. For example if a particular country, felt that abatement would harm it, it would not abate at all, even though the net global effect of its abatement would be the same as everyone else's. But in the special case where all countries are identical except for size, so that the only problem is the free-rider problem, then a global price target induces perfect cooperation. This is why the proof requires the proper scaling of the cost and benefit parameters to the size of the country.

6.2 A Rich-and-Poor Game with a Price Target

Equation (p10) shows that country j will vote for global target price $P_j = c_j\ b_j\ \Sigma\ (\ 1/c_j\ )$. So if two countries are the same size and have the same cost function, but view the benefits differently, meaning they have different b_j's, then they will choose different prices. In fact, if even one country has a $b_j = 0$ it will vote for a price of zero with devastating consequences for the game as specified.[3]

6.3 Using a Green Fund to Motivate a Price Target

As shown, in pure (no-free-riding) RP games the size of a country will not affect its choice of price (unlike in the PG Game), but other factors will. This can cause an uncooperative outcome. Consequently, it can be worthwhile to pay countries to accept a higher price. When the choice of a low price is due to poverty or the view that the country is not responsible for the problem, such a payment may seem more than reasonable. In fact such payments are widely proposed (though usually not linked to a requirement to cooperate) and are often embedded in a concept called a Green Fund.

Since low emissions are correlated with both low income and low responsibility for the climate problem, and since rewarding low emissions is an ideal incentive, emissions per capita is an ideal variable to reward when encouraging acceptance of a high target price. As a clear, but rather stylized, example of how Green Fund payments can encourage cooperation, the next game assumes that low-emission countries consider abatement costs more onerous in inverse proportion to the country's emissions per capita. However Green-Fund payments remain unaffected by this effect. This is called the RP#1 economy.

Table 16. Solution of the Public-Goods-Rich-and-poor#1/Green-Fund Game

Definition of the PG-RP#1/GF Game		
$\text{NB}_\text{j} = b_j A - (\text{e}/e_j) c_j A_\text{j}^2 - g\,(e_j - \text{e})\ n_j\ P$	The last term is the Green Fund payment	(q1)
$E_j = e_j\, n_j$	(cn12)	(q2)
$bj = b\, E_j$	(cn13)	(q3)
$cj = c\ /\ E_j$	(cn14)	(q4)
$\text{k} = \Sigma\,(1/c_j)$	(cn7)	(q5)
$g = b\,k/2$	This completes the Green-Fund policy	(q6)
The Carbon Market		
$P = 2\ c_j A_\text{j}$	(cn9)	(q7)
$A_\text{j} = P\ /\ 2\ c_j$	(q7)	(q8)
$A = \Sigma\, A_\text{j} = k\,P/2$	Total abatement from (q8)	(q9)
Solving for preferred P		
$\text{NB}_\text{j} = b_j\, k\, P/2 - ((e/e_j)/4c_j)\ P^2 - g\,(e_j - \text{e})\ \text{nj}\ P$	(q1), (q8) and (q9)	(q10)
$\text{d NB}_\text{j}\ /\ dP = 0$	Optimization	(q11)
$b_j\, k/2 - (e/e_j)(1/2c_j)\ P - g\cdot(e_j - \text{e})\ n_j = 0$	Differentiate	(q12)
$(e/e_j)(1/2c_j)\ P = b\,k/2 - g\cdot(e_j - \text{e})\ n_j$		(q13)
$(e/e_j)\ P\ /\ 2c_j = b_j\ k/2 - b\,k\cdot(1 - e/e_j)\ E_j\ /\ 2$	(q2) and q(6)	(q14)

[3] Incidentally, it may seem odd that when c_j increases, P_j will increase. This is because with a higher c_j, less will be abated, and since cost increases with the square of abatement, abatement costs are actually reduced. In the limit, when c_j becomes infinite, the country will not respond at all to price, and so it will suffer no cost from carbon pricing. In this case, raising the global price provides benefit at no cost.

$(e/e_j)\ P/2c_j\ = b_j\ k/2 - b_j k\cdot(1 - e/e_j)/2$	(q3)	(q15)
$(e/e_j)\ P/2c_j\ =\ b_j\ k\cdot(e/e_j)/2$		(q16)
$P/2c_j = b_j\ k/2$		(q17)
$P = c_j\ b_j\ k$		(q18)
$P\ =\ B$	See (p10) through (p13). This shows that P is the same for all countries.	(q18)

The result is that all countries vote for the same global target price voluntarily. Those with low emissions per capita do so because a higher price means they will receive greater Green Fund payments.

6.3.1 Optimality?

As seen in (q18) each country's abatement is the same as the optimal value found for the rich-and-poor game in (p13). However, the game has been changed, so this may no longer be optimal. However, in a limited sense it is optimal as can be shown with the following Lemma.

Lemma 1: If all countries vote for the same price then that price maximizes global net benefit within the constraints of the game's rules.

Proof: First we define the global net benefit to be the sum of the individual country's net benefits. NB = NB_1 + NB_2 + … NB_N. Second, note that the price-target game requires all countries to implement the same price of abatement. Let P^* be the price that all countries voted for. We need to show that NB(P^*) ≥ NB(P) for all P. But the fact that each country chose to vote for P^* means this is true for each country's NB_j individually. If each element of a sum is greater, then the sum itself must be greater.

Lemma 1 tells us that the outcome of the Green-Fund game is best within the set of policies that result in a uniform price of abatement, such as cap-and-trade, a global price target, or any dictated uniform price. Generally it is argued that an optimal uniform price is best because it results in efficient abatement. However this argument fails to consider the redistribution of wealth and effort that can accompany such a policy. If a policy changes the distribution of wealth or effort, nothing can be said about its optimality without consulting a social-welfare function, which we define to be the global net benefit function. Since efficiency does not consider this function, but only considers the carbon market, efficiency does not imply optimality. The social optimum is derived as follows:

Table 17. Solving for the Optimal Social Outcome in a Public-Goods, Rich-and-poor#1 Economy

GNB $= B\,A - (e/e_j)[\,c_1A_1^2 + c_2A_2^2 \ldots c_NA_N^2\,]$	Global net benefit. Note that Green Fund payments cancel out.	(r1)
$B = \Sigma\, b_j$	Definition (cn4)	(r2)
$d\,GNB / d\,P_j = 0$	Optimize one country's price. Here we assume that the rules of the game don't hold, and each country can set its own price and determine its own abatement	(r3)
$P_j = 2\,c_j A_j,\quad A_j = P_j/2c_j$	(cn9)	(r4)
$dA/dP_j = 1/2c_j$	(r4) Only A_j is effected by P_j	(r5)
$0 = B/2c_j - (e/e_j)\,2\,c_j A_j\,(dA_j/dP_j)$	Differentiate (r1) and use (r5)	(r6)
$0 = B/2c_j - (e/e_j)\,2\,c_j (P_j/2c_j)\,(1/2c_j)$	(r4), (r5)	(r7)
$0 = B/2c_j - (e/e_j)\,(P_j/2c_j)$	(r7)	(r8)
$B = (e/e_j)\,P_j$	(r8)	(r9)
$P_j = (e_j/e)\,B$	B is the optimal price in the basic game. The optimal price now differs between countries.	(r10)

Because the low-emission countries see abatement as more costly than the market cost of abatement (perhaps because they see the inconvenience of abatement as unfair) and because this effect is not balanced by a view that Green-Fund payments are worth more than their market value, the socially optimum abatement patterns requires less abatement-per-capita by low-emission countries. This warns that efficiency is not the same as social optimality.

In spite of this warning, we will now move to a model economy, rich-and-poor economy #2, in which low-emission countries view Green-Fund payments through the same magnifying lens as abatement costs. This will tend to align efficiency and optimality. Interestingly, we will find a policy which is nearly efficient and socially superior to the most-popular efficient outcome.

7 Price Targets and Green Funds in a More Realistic Setting

In real applications it would be useful to employ a simple formula for the Green Fund and have it produce a reasonably good outcome, even though it is not optimal. This section shows that such a hope is not unrealistic. We will consider a public-goods game in which poor countries value abatement costs and Green-Fund payments disproportionately more than do rich countries. This is the same assumption used in the cooperative rich-and-poor game (m8).

However, this assumption will be applied only to countries with below-average emissions per capita. For above-average emitters we will assume normal evaluation of costs and benefits. In such a game, low-emission countries will need Green Fund payments to get them to vote for the optimal price target, but high-emission countries will not. But if the rich countries are not part of the Green-Fund mechanism, the Green Fund will have no funds. Hence it is necessary to require contributions to the Green Fund from high-emission countries and accept the fact that this will cause them to vote for a sub-optimal price target. Moreover, the simple Green-Fund formula will be incorrect for the low-emission countries. A more complex formula could motivate them optimally, but we will assume it is politically infeasible and stick with this simple formula for both rich and poor countries:

Payments to the Green Fund $= g \cdot (e_j - e)\, n_j P = g \cdot (E_j - E)\, P$

In other words a high emission country must pay for its excess emissions at a price that is g times the global price target.[4] (The Green-Fund payments will turn out to be quite small.) Low-emission countries receive payment in proportion to how-far their emission are below the global per-capita average. This formula is not only simple, but also revenue neutral. It collects exactly as much as it disperses.

7.1 Defining the "Realistic" Game

The first layer of the realistic game is the basic public goods model in which all countries are identical except for size. To this is added the RP economy #2 factor for low-emission countries. This means that such countries see their abatement costs and Green-Fund payments magnified by (e/e_j). This factor does not, however, affect their perception of climate benefit. The result is that, without a Green Fund, they would abate less than in the standard public goods model.

To this economic model we add a global price target mechanism and a Green-Fund incentive. Since Green-Fund payments are proportional to the adopted target price, low-emission countries have an extra incentive to vote for a high target price.

All countries have an incentive to set the Green-Fund's policy parameter g to a reasonable level. If, for example, low-emission countries were to set it very high, then high-emission countries would vote for a low target price, and since the lowest vote wins, this would reduce payments to low-emission countries. Nonetheless, the determination of g is not part of the game, and the outcomes presented below are based on an optimal choice of g. This maximizes the minimum P that's voted for, and thereby maximizes abatement. In spite of this, abatement is suboptimal.

[4] Although e_j will be changed by the policy, the value used for the Green-Fund payments will be defined to be fixed in this game.

Table 18. Finding Countries' Strategies in the "Realistic" Game

Finding the P that Low-e countries will vote for		
$NB_j = b_j A - (e/e_j)\,[\,c_j\,A_j^2 + g_j\,P\,]$	(me8)	(s1)
$g_j = g\cdot(e_j - e)\,n_j$	(cn15)	(s2)
$A = kP/2$, and $c_j\,A_j^2 = P^2/(4c_j)$	Sum of (g2), and (g2) again	(s3)
$NB_j = b_j\,k\,P/2 - (e/e_j)\,[\,P^2/(4c_j) + g_j P\,]$	(s1), (s3)	(s4)
$dNB_j/dP = 0$ implies P_j^*	Condition for optimal voting	(s5)
$b_j\,k/2 - (e/e_j)[\,P/2c_j + g_j\,] = 0$	Differentiate	(s6)
$(e/e_j)P/(2c_j) = b_j k/2 - (e/e_j)\,g\cdot(e_j - e)\,n_j$	(s6)	(s7)
$P = (e_j/e)\,c_j b_j k - 2\,c_j\,g\cdot(e_j - e)\,n_j$	(s7)	(s8)
$P = (ej/e)\,c\,b\,k - 2\,(c/E_j)\,g\cdot(1 - e/e_j)\,E_j$	(cn13), (cn14)	(s9)
$P = (ej/e)\,c\,b\,k - 2\,c\,g\cdot(1 - e/ej)$	Best choice of P given Green-Fund g	(s10)
Finding the P that High-e countries will vote for		
$NB_j = b_j\,A - c_j A_j^2 - g_j P$	No rich-and-poor effect for high-e countries	(s11)
$NB_j = b_j k P/2 - P^2/(4c_j) + g_j P$	(s3)	(s12)
$0 = b_j k/2 - P/(2c_j) - g_j$	Differentiate with respect to P	(s13)
$P/(2c_j) = b_j\,k/2 - g\cdot(e_j - e)\,n_j$	(cn15)	(s14)
$P = c_j b_j k - 2\,c_j\,g\cdot(ej - e)\,n_j$	(s14)	(s15)
$P = c\,b\,k - 2\,(c/E_j)\,g\cdot(1 - e/e_j)\,E_j$	(cn13), (cn14)	(s16)
$P = c\,b\,k - 2\,c\,g\,(1 - e/e_j)$	Best choice of P given Green-Fund g	(s17)

7.2 Outcomes of the "Realistic" Game

The outcome of this game in a three-country global economy is displayed in Table 19. The value of g (0.036), the Green-Fund parameter, was set to produce the highest minimum voted global target price. The result is a target price that is not far below the optimal target price, of $30/ton, shown in Table 20. The Green-Fund charge for above-average emissions is $0.95 per ton.

Table 19. The outcome for the "Realistic" Economy with a Simple Green Fund

Country	n_j	e_j	Voted P_j	Target P_j	A_j	A_j %	Benefit	Cost	G.Fund	NB
	Billions	t/cap./yr	$/ton	$/ton	B ton			$/cap./yr		
U.S.	0.3	18	$26.04	$26.04	0.94	17.4%	$28	–$12	–$4	$12.21
China	1.2	5	$30.00	$26.04	1.04	17.4%	$31	–$14	$0	$17.69
India	1.0	1.1	$26.04	$26.04	0.19	17.4%	$6	–$2	$4	$11.30
Total/Avg.	2.5	5			2.17		$65	–$28	$0	$41.19

Table 20. Efficient Outcome for "Realistic" Economy

Country	n_j Billions	e_j ton/cap./yr	Opt. P $/ton	A_j B ton	A_j %	Benefit $/cap./yr	Cost $/cap./yr	NB $/cap./yr
U.S.	0.3	18	$30	1.08	20%	$32.40	-$16.20	$16.20
China	1.2	5	$30	1.20	20%	$36.00	-$18.00	$18.00
India	1.0	1.1	$30	0.22	20%	$6.60	-$3.30	$3.30
Total/Avg.	2.5	5		2.50		$75.00	-$37.50	$37.50

The efficient outcome in Table 20 ignores the rich-and-poor aspect of the economy and sets $P = B$, the optimal value in a standard public-goods economy. But how should this be interpreted when the low-e countries evaluate their net benefits according to the rich-and-poor#2 model?

First, within the rich-and-poor#2 model, without a GF incentive, poor (low-e) countries will choose a low level of abatement. But that makes marginal abatement cheap from a market perspective. The high-e countries can take advantage of this by paying the cost of abatement as it is viewed by the low-e countries. This is not a problem because, although low-e countries over-value the cost of abatement, they also overvalue Green-Fund payments. So low-e abatement can be carried out by high-e countries just as if there were no rich-and-poor effect. If they pay just enough to keep the low-e country's net benefit constant then we have the outcome shown in Table 21.

But what if the high-e countries pay more? In fact it seems likely that low-e countries would capture some infra-marginal rents. This area has not yet been investigated except for the numerical calculations reported in Table 19. But, since poor countries derive more welfare from revenue that do rich countries, if such transfers occur they will increase global welfare.

Table 19 shows the outcome of using a global-price-target mechanism and a simple Green-Fund incentive with the best green fund rate, which is $0.95/ton of emissions over or under what would have been emitted at the global average rate of emissions per capita. Note that because the policy mix is sub-optimal, the voted target price is 13% too low at $26 instead of $30/ton. The result is an abatement of 2.17 billion tons instead of 2.5 billion. Interestingly, the global net benefit, which is the measure of social welfare, is slightly greater than the value found it Table 20.

Finally, it is important to note the magnitudes. This example describes a policy that is comparable in strength to what Europe has adopted and to what the United States is considering. Moreover, the United States ends up paying for India's abatement costs twice over. Also, it should be noted that the cost of abatement is roughly twice as high as assumed by Nordhaus. In spite of this, the full cost to the United States, of abatement and Green-Fund payments, is only $16 per person per year. That's roughly fifty times less than the cost of a cell phone.

8 Relevant Policy Literature

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