Global Climate Games:
How Pricing and a Green Fund Foster Cooperation

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Technical Appendix

(paper and spreadsheet are available at www.global-energy.org/lib/11-01)

This appendix derives the results reported in the associated paper, including the derivation of formulas used in the spreadsheet that computes the equilibria of the example games reported in Table 2, 3, and 4.

Numerical Checking. The spreadsheet allows many of these calculations to be checked. Most of these derivations find a formula that optimizes a function, $\mathrm{f}(X, a, b)$. The result is a new function $\mathrm{x}(a, b)$. To check that $\mathrm{x}(.,.)$ performs as claimed, the spreadsheet allows easy comparison of $\mathrm{f}(\mathrm{x}(a, b)+\Delta, a, b)$ with $\mathrm{f}(\mathrm{x}(a, b), a, b)$. Any Δ other than zero should reduce the value of $\mathrm{f}(\mathrm{x}(a, b), a, b)$. All of the checks are successful.

1. USING ABATEMENT TO ANALYZE EMISSIONS

We begin with the optimal climate policy and the global public goods game for a world with N countries. For the examples, each country, j, has a net-benefit function (aside from carbon trading) as follows:

$$NB_j = b_j A - c_j A_j^2, \text{ where } A = \sum_{i=1}^{N} A_i \qquad (1.1)$$

Each country's abatement, A_j, is the difference between its "business as usual," emissions level, E_j^U, and its actual emissions level, E_j. Although the business-as-usual value is hard to know, the cost of reducing emissions cannot be formulated without knowing it. So, working with abatement, which cannot be directly measured, instead of actual emissions, which is measurable, does not cause any additional difficulty, and it is analytically convenient. (See also sections 4 and 5 below.) The assumption of a quadratic cost-of-abatement function is standard, and a linear climate benefit function, $b_j A$, should be reasonably accurate in the plausible range of near-term abatement values. As abatement grows, we expect the marginal benefit of abatement to decrease, as assumed in Proposition 1b.

2. OPTIMAL ABATEMENT

Optimal abatement requires choosing the set of abatements, $\{A_j\}$, to maximize $\sum_{i=1}^{N} NB_i$. (All summations run over the set of all countries.) This implies that the abatements must satisfy the following first order conditions:

$$\frac{d}{dA_j}\left[A \sum b_j - c_j A_j^2\right] = 0. \qquad (2.1)$$

(There is no need to sum over costs, because other country's costs are not affected by A_j.) This simplifies to,

$$\sum b_j = 2c_j A_j, \text{ or } A_j = P^O/2c_j, \text{ where } P^O = \sum b_j, \qquad (2.2)$$

where P^O is the optimal global price to pay for "carbon" abatement or to charge for carbon emissions. (Note the use of $dA/dA_j = 1$.)

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3. THE GLOBAL-PUBLIC-GOODS GAME

In the global-public-goods game, countries choose their abatement considering only their own benefit and cost, and they assume that other countries do not react to their choice—this defines a Nash equilibrium. (With linear benefits, this last condition is irrelevant.) This produces an inefficient equilibrium in which each country, in effect, has a different price of carbon. The Nash equilibrium conditions for this game are:

$$b_j = 2c_j A_j, \text{ or } A_j = b_j/2c_j. \tag{3.1}$$

The effective carbon prices are $P_j = b_j$. So, from Equation (2.2), $P^O = \sum P_j$.

4. THE GLOBAL CAP-AND-TRADE GAME

The global cap-and-trade game, adds the gains from selling carbon credits to the net-benefit equation (1.1).

$$NB_j = b_j A - c_j A_j^2 + P \cdot (A_j - T_j), \text{ where } A = \sum_{i=1}^{N} A_i \tag{4.1}$$

In this game the strategic variable that countries control changes from A_j to T_j, the country's abatement target. If the target is met domestically (without trading credits) the target would result in $E_j^U - T_j$ emissions, so the country issues that many internationally valid carbon credits. Note that a larger abatement target, means fewer credits issued. It does no harm, and it simplifies the presentation, to ignore business-as-usual emission, actual emissions, and carbon-credits issued, and to work instead with only the summary variables, abatement and the abatement target. This simply requires noting that if abatement is greater than the target there will be surplus credits in an amount equal to the difference, which will be sold outside the country at, P, the global price of carbon. This explains the third term in the net benefit formula.

There are two sets of first order conditions for the Nash equilibrium of the cap-and-trade game: one for the strategic variable and one for the carbon markets.

$$\frac{dNB_j}{dT_j} = 0 \tag{4.2}$$

$$2c_j A_j = P \tag{4.3}$$

Note that total abatement must equal the total of all abatement targets, so

$$A = \sum A_j = \sum T_j, \text{ and } \frac{dA}{dT_j} = 1. \tag{4.4}$$

We begin finding the equilibrium values of T_j by expanding Equation (4.2) using Equation (4.1).

$$b_j \frac{dA}{dT_j} - 2c_j A_j \frac{dA_j}{dT_j} + P \cdot \left(\frac{dA_j}{dT_j} - 1\right) + \frac{dP}{dT_j}\left(A_j - T_j\right) = 0 \tag{4.5}$$

Using Equations (4.3) and (4.4) this simplifies to:

$$b_j - P + \frac{dP}{dT_j}\left(A_j - T_j\right) = 0 \tag{4.6}$$

Next we solve Equation (4.3) for A_j and sum over all j to find:

$$\sum A_j = \sum\left(P/2c_j\right)$$

$$A = P \sum\left(1/2c_j\right)$$

$$P = A/\sum\left(1/2c_j\right) \tag{4.7}$$

Next differentiate with respect to T_j, then use Equation (4.4) and define k.

$$\frac{dP}{dT_j} = \frac{dA}{dT_j} / \sum\left(\frac{1}{2c_j}\right) = 2/\sum \frac{1}{c_j} \triangleq 2/k \tag{4.8}$$

Now substitute this in to Equation (4.6), sum over j, and notice that the term in parenthesis sums to zero. This gives,

$$\sum b_j - \sum P + (2/k) \cdot (0) = 0 \quad (4.9)$$

Divide by n to find the cap-and-trade global carbon price.

$$P^* = \frac{1}{N}\sum b_j \quad (4.10)$$

From Equation (4.3) find $$A_j = P^*/2c_j \quad (4.11)$$

Substituting Equation (4.8) into Equation (4.6) gives

$$T_j = A_j + (b_j - P^*)k/2 \quad (4.12)$$

This completes the solution of the Nash cap-and-trade game defined by payoff Equation (4.1) with strategic variables, T_j. This solution is used in the spreadsheet found at www.global-energy.org/lib/11-01 to compute the values in Table 2 of the paper. We now prove Proposition 1, which does not rely on the quadratic cost function of Equation (4.1), and which, in part 1b, does not rely on the linear benefit function.

5. PROPOSITION 1: THE CAP-AND-TRADE PRICE IS N–TIMES TO LOW

Proposition 1 shows that global cap-and-trade, in which caps are selected voluntarily, and not imposed on countries by some world climate authority, cannot approach the optimal outcome. Part (a) compares the carbon price under cap and trade directly to the optimal value and find it to be N times lower (N being the number of countries). However, this proof requires that the abatement benefit function be linear. If marginal benefits decline with abatement, then to the extent they do, the optimal global carbon price is reduced. So the only reason that cap-and-trade price might exceed $1/N$ times the optimal price is because that price has been reduced by the success of the optimal policy.

Part (b) compares the cap-and-trade price with the average public-goods price and overcomes the ambiguity caused by the abatement level in Part (a) by assuming that the cap-and-trade policy has some success and reduces abatement relative to the public-goods level of abatement.

Proposition 1. In the global cap-and-trade game, with global carbon price, P^*,

a) If the marginal benefit of abatement is constant, then P^* equals $1/N$ times the optimal price.

b) If the marginal benefit of abatement decreases with abatement and cap-and-trade serves to increase global abatement, then P^* is less than the un-weighted, country-average, public-goods price.

Proof:

The global public goods game is defined to be one in which countries do not tamper with their domestic price of carbon, but instead allow the global price to prevail. (This also means all abatement projects must be cost effective at the global carbon price.) Countries are presumed to know their "business as usual," emissions level, E_j^U, so that they can compute abatement costs. Their strategic variable is their emissions cap, E_j^C, and they issue internationally valid carbon credits up to this cap. Private companies interacting with the global carbon market determine each country's actual emissions level, E_j. To analyze this game we define

$$\text{The abatement target, } T_j = E_j^U - E_j^C, \text{ and actual abatement, } A_j = E_j^U - E_J.$$

Given these definitions we define the global cap-and-trade game as follows

$$\text{Country } j \text{ chooses } T_j \text{ to maximize } NB_j = b_j A - C_j(A_j) + P \cdot (A_j - T_j), \quad (5.1)$$

where $A = \sum A_j$ is total global abatement,

$b_j A$ is country j's climate benefit function,

$C_j(A_j)$ is the county's cost of abatement, and

P is the global price of carbon credits.

The efficiency of the carbon-credit markets and the private sector assures that

$$\frac{dC_j(A_j)}{dA_j} = P. \quad (5.2)$$

Since countries can choose their own targets, there is no reason for a country to issue more credits than its target warrants, and since we assume there is no cheating (until the next section):

$$A = \sum A_j = \sum T_j, \text{ and } \frac{dA}{dT_j} = 1. \tag{5.3}$$

The first-order Nash condition requires that the partial derivative of NB_j with respect to T_j is zero (this derivative holds other T_k constant). Applying this condition to Equation (5.1) leads to

$$b_j \frac{\partial A}{\partial T_j} - \frac{dC_j(A_j)}{dA_j}\frac{\partial A_j}{\partial T_j} + P \cdot \left(\frac{\partial A_j}{\partial T_j} - 1\right) + \frac{dP}{dA}\frac{\partial A}{\partial T_j}\left(A_j - T_j\right) = 0 \tag{5.4}$$

Note that with an efficient market, the global carbon price depends only on total global abatement, A, which allows the dependence of P on T_j to be expressed indirectly as shown in Equation (5.4). Using Equation (5.2) and Equation (5.3) leads from (5.4) to:

$$b_j - P + \frac{dP}{dA}\left(A_j - T_j\right) = 0 \tag{5.5}$$

Summing over all N countries and using Equation (5.3) leads to:

$$\sum b_j = \sum P \tag{5.6}$$

Finally, dividing by N gives the global cap-and-trade price of carbon, P^*, and Equation (2.2) shows how this relates to the optimal global price.

$$P^* = \frac{1}{N}\sum b_j = \frac{1}{N} P^O \tag{5.7}$$

This concludes the proof of Proposition 1a.

For proposition 1b, replace $b_j A$ with $B_j(A)$ in the net-benefit Equation (5.1). Let A^* be total abatement in the cap-and-trade equilibrium and A^G be total abatement in the public goods equilibrium. Assume cap-and-trade increases abatement, so that $A^* > A^G$, and assume that the total marginal benefit decreases with increasing abatement. Then by following the same path as in equations (5.1) through (5.7) we find:

$$P^* = \frac{1}{N}\sum \frac{dB_j(A)}{dA}\bigg|_{A^*} < \frac{1}{N}\sum \frac{dB_j(A)}{dA}\bigg|_{A^G} = \text{Avg}(P^G), \tag{5.8}$$

where $\text{Avg}(P^G)$ is the un-weighted country average of public-goods-game prices. Hence,

$$P^* < \text{Avg}(P^G). \tag{5.9}$$

This concludes the proof of Proposition 1b.

6. CHEATING IN THE GLOBAL CAP-AND-TRADE GAME (THEORY)

As discussed in the paper, under global cap and trade, countries can cheat by subsidizing fossil fuel. In effect, this reduces their domestic price of carbon relative to the global price of carbon, allowing them to spend less on abatement. The paper claims that if all countries optimize this form of cheating, they will all end up abating exactly the same amount as they would in the public-goods game, and an example is given. The general result is stated here as Proposition 2. That result is proven in this section, and the next section uses it to solve for the equilibrium of the example game reported in Table 3 of the paper.

Proposition 2. In the global cap-and-trade game with emission subsidies, countries will (a) choose the same emission levels they choose in the public goods game, although (b) they will continue to trade carbon credits.

Proof:

This result has been proven by Godal and Holstmark (2010), but a proof is included here for the reader's convenience. The global cap-and-trade game with cheating that uses emissions subsidies is treated as a new game in which the use of emissions subsidies is allowed. That game will be called the ES game, and its strategic variables are T_j and s_j, the subsidy rate. This game's net-benefit (payoff) function is the same as that of the global cap and trade game, which is:

$$NB_j = B_j(A) - C_j(A_j) + P \cdot (A_j - T_j). \tag{6.1}$$

The cost-of-abatement function does not include the subsidy for emissions, because the net benefit applies to the country and the subsidy is a transfer payment within the country and, as such, has no direct net cost. The private abatement market however includes the subsidy in its optimization. The cost of abatement is a net cost to abaters that includes monetary costs and lost utility minus the savings from reduced fuel costs. When fossil-fuel emissions are subsidized, abatement results in less fuel-cost savings and consequently in higher net abatement costs, as shown in Equation (6.2).

$$\text{Abatement cost including fossil subsidy } = C_j(A_j) + s_j A_j, \tag{6.2}$$

where s_j is the emission subsidy rate of country j. Consequently the new set of carbon-market equilibrium conditions are as follows:

$$\frac{dC_j(A_j)}{dA_j} + s_j = P, \tag{6.3}$$

This relationship allows the country to use s_j to control its domestic abatement level, A_j. Because of this, A_j can be thought of as the new strategic variable in place of s_j, as can be seen from the first-order condition of the new strategic variable s_j, which follows:

$$0 - \frac{dC_j(A_j)}{dA_j}\frac{\partial A_j}{\partial s_j} + P \cdot \left(\frac{\partial A_j}{\partial s_j} - 0\right) + \frac{\partial P}{\partial A_j}\frac{\partial A_j}{\partial s_j}\left(A_j - T_j\right) = 0 \tag{6.4}$$

Note that the first term is zero because A is determined by the abatement targets and unaffected by s_j. Equation (6.4) shows that $\partial A_j / \partial s_j$ cancels out. The simplified condition is:

$$P - \frac{dC_j(A_j)}{dA_j} + \frac{\partial P}{\partial A_j}\left(A_j - T_j\right) = 0 \tag{6.5}$$

Now, suppose that country j did not use subsidies but was able to control A_j directly while maintaining the domestic efficiency of this abatement. All other countries are still assumed to control the strategic variables T_i and s_i. In this case the first-order condition for A_j would be:

$$0 - \frac{dC_j(A_j)}{dA_j} + P \cdot (1 - 0) + \frac{\partial P}{\partial A_j}\left(A_j - T_j\right) = 0 \tag{6.6}$$

This is the same as Equation (6.5), which confirms the assertion that county j can be analyzed as optimizing T_i and A_i, while all other countries hold T_i and s_i constant. (Note that it is incorrect to think of all countries changing to the strategic variables T_i and A_i simultaneously because total abatement must sum to total abatement targets, which makes it impossible to change the value of just one of these variables.) The insight that subsidies allow a country to control T_i and A_i independently, is the key to understanding this result.

Now take the partial derivative of Equation (6.1) with respect to T_j to find the other first order condition:

$$\frac{dB_j(A)}{dA}\frac{\partial A}{\partial T_j} - \frac{dC_j(A_j)}{dA_j}\frac{\partial A_j}{\partial T_j} + P \cdot \left(\frac{\partial A_j}{\partial T_j} - 1\right) + \frac{\partial P}{\partial T_j}\left(A_j - T_j\right) = 0 \tag{6.7}$$

Note that since A_j and T_j are the strategic variables, $\partial A_j / \partial T_j = 0$. Also recall that $\partial A / \partial T_j = 1$. So Equation (6.5) simplifies to

$$\frac{dB_j(A)}{dA} - P + \frac{\partial P}{\partial T_j}\left(A_j - T_j\right) = 0 \tag{6.8}$$

Now note that Country j can only effect the market for carbon credits through its supply of credits to the global market and that supply is $\left(A_j - T_j\right)$, consequently changes in A_j and T_j have opposite effects on P. That allows Equations (6.5) and (6.8) to be summed to produce:

$$\frac{dB_j(A)}{dA} = \frac{dC_j(A_j)}{dA_j} \tag{6.9}$$

Since $\partial A/\partial A_j = 1$, this is equivalent to the condition used by each country in the public good game:

$$\frac{dB_j(A)}{dA}\frac{\partial A}{\partial A_j} = \frac{dC_j(A_j)}{dA_j} = P_j^G \; . \tag{6.10}$$

Hence each country's abatement will be the same as in the public goods game, which proves Proposition 2(a).

Now consider the possibility of international carbon trading under the ES game, part (b) of Proposition 2. Because there is only one global carbon price under cap-and-trade, most countries will find their effective public-goods price, P_j^G, either higher or lower than the global carbon price. If we imagine starting at an equilibrium in which each country sets its target equal to its public goods abatement level, then countries with $P_j^G < P^*$ will find it to their advantage to reduce their abatement target and sell more carbon credits, even though this reduces the total global abatement, this is because their value per ton of selling credits is P^*, but their value per ton of reducing global emissions is only P_j^G. Similarly, countries with $P_j^G > P^*$ will wish to raise their targets. These two effects cancel out with regard to abatement, but they result in a flow of funds from countries that favor a high global price of carbon to countries that favor a low global price.

7. CHEATING IN THE GLOBAL CAP-AND-TRADE GAME (THE EXAMPLE IN TABLE 3)

The net benefit function used in this example takes the form shown in Equation (4.1). As in the proof above we will treat A_j and T_j as the strategic variables for country j (but not for other countries when analyzing country j). This implies that when differentiating with respect to T_j, A_j must be held constant. This simplifies the partial derivative of NB_j with respect to T_j as follows:

$$b_j - P + \frac{dP}{dT_j}\left(A_j - T_j\right) = 0. \tag{7.1}$$

From Equation (6.3) and the functional form for abatement cost, we have

$$P = 2c_jA_j + s_j \tag{7.2}$$

In the example, there are only two countries, so when country j changes T_j and holds A_j constant, the abatement of the other country must change abatement because total abatement equals the sum of the targets and the other country is holding constant s_j and T_j. Consequently, for $k \neq j$ we have:

$$\frac{dP}{dT_j} = 2c_k \tag{7.3}$$

To understand this, recall that, when country j changes only T_j, it does this by adjusting s_j to keep A_j constant, but then the other A_k must change and this is possible because the other country is not controlling A_k, but rather, s_k. Substituting Equation (7.3) into (7.1) gives us two equations, and using Equation (5.3) gives us one more for a total of three equations for three unknowns, P, T_1 and T_2.

$$P = b_1 + 2c_2(A_1 - T_1) \tag{7.4}$$

$$P = b_2 + 2c_1(A_2 - T_2) \tag{7.5}$$

$$T_1 + T_2 = A_1 + A_2 \tag{7.6}$$

Solve for the term in parenthesis in Equations (7.4) and (7.5) and then add these together to find:

$$(P - b_1)/2c_2 + (P - b_2)/2c_1 = (A_1 - T_1) + (A_2 - T_2) \tag{7.7}$$

Notice that the right side is zero because of Equation (7.6) and solve for P to find:

$$P = \left[\sum b_j/2c_k\right]/\left[\sum 1/2c_j\right] \tag{7.8}$$

Given P, the two targets are easily found from Equations (7.4) and (7.5) since the A_j are known to be equal to their value in the Public-Goods game. The subsidies can be found from Equation (7.2). The spreadsheet allows for this solution to be checked numerically by testing variations in the strategic variables and observing the corresponding country's net benefit.

8. THE GREEN-FUND GAME

The first step of the Green-Fund game requires the average country ("China") to choose a value for G, the Green-Fund strength coefficient. To do this rationally, China must know how all three countries will vote when the global price target, P^T, is chosen. Their preferences will depend on the Green-Fund parameter. So, the first step is to find $P^V(G)$, the price-target voting function for country j. This must be determined from the net-benefit function,

$$NB_j = b_j A - c_j A_j^2 + G \cdot \Delta E_j \cdot P, \text{ where } \Delta E_j = (e - e_j)\, n_j \tag{8.1}$$

As explained in the paper, e is global average emissions per capita, e_j is the country's emissions per capita, and n_j is its population. The private abatement market will set marginal cost equal to the global price:

$$2c_j A_j = P, \text{ or } A_j = P/2c_j \tag{8.2}$$

Summing over all countries and defining k gives:

$$A = \sum A_j = (P/2) \sum \frac{1}{c_j} \triangleq kP/2 \tag{8.3}$$

Substituting from Equations (8.2) and (8.3) and into (8.1) gives

$$NB_j = b_j kP/2 - P^2/4c_j + G \cdot \Delta E_j \cdot P \tag{8.4}$$

Differentiating with respect to P and setting this to zero gives the first-order condition for a country's preferred P, which is the price it will vote for. The next step solves for P_j^V.

$$b_j k/2 - P_j^V/2c_j + G \cdot \Delta E_j = 0 \tag{8.5}$$

$$P_j^V = c_j b_j k + 2c_j \cdot G \cdot \Delta E_j, \text{ where } k = \sum \frac{1}{c_j} \tag{8.6}$$

When China adjusts G to find the value that maximizes the minimum P_j^V, it finds that the U.S. and India will cast the low votes, and the higher one goes, the lower the other goes. So the problem becomes finding the G which makes the U.S. and India vote for the same price target. Denoting these countries by $j = 1$ and $j = 3$, the problem is solved as follows:

$$c_1 b_1 k + 2c_1 \cdot G \cdot \Delta E_1 = c_3 b_3 k + 2c_3 \cdot G \cdot \Delta E_3 \tag{8.7}$$

$$2(c_1 \cdot \Delta E_1 - c_3 \cdot \Delta E_3)G = (c_3 b_3 - c_1 b_1)k \tag{8.8}$$

$$G^v = \frac{(c_3 b_3 - c_1 b_1)k}{2(c_1 \cdot \Delta E_1 - c_3 \cdot \Delta E_3)} \tag{8.9}$$

This is the value China will vote for, and the global target price will be given by Equation (8.6) and the coefficients for either the U.S. or India.

The rest of the Green-Fund equilibrium values are trivial to compute because the only equilibrium conditions to solve are those for the private market which determines A_j's according to Equation (8.2). Benefits, costs and Green-Fund payments are then simply computed from the three parts of the net-benefit formula.